Techniques for Proof Compression

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Motivations for Proof Compression

- Sat/SMT-solvers, ATPs, proof assistants. . .
  - best techniques to find proofs do not necessarily find the best proofs
  - proofs can be redundant
- Proof compression techniques may lead to:
  - smaller proof libraries
  - faster proof checking
  - smaller unsat cores
  - better interpolants
  - easier exchange of knowledge
  - discovery of interesting mathematical definitions and lemmas

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Techniques for Proof Compression

- Sequent Calculus
  - *Cut-elimination*
  - *Cut-introduction*
- Natural Deduction
  - *Allowing contextual inferences*
- Propositional Resolution
  - *Recycle Pivots (with Intersection)*
  - *Lower Units*
  - Reduce&Reconstruct
  - Split

Natural Deduction

$$\frac{}{\Gamma, A \vdash A}\ \textit{axiom}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B}\ \rightarrow_I$$

$$\frac{\Gamma \vdash A \rightarrow B \qquad \Gamma \vdash A}{\Gamma \vdash B}\ \rightarrow_E$$

Figure: The natural deduction calculus **ND**

Double negation elimination axiom schema:

$$dne : \neg\neg F \to F$$

Deriving $(A \to B) \to C$ from $(\neg\neg A \to B) \to C$ in **ND**:

$$\dfrac{(\neg\neg A \to B) \to C \vdash (\neg\neg A \to B) \to C \qquad \dfrac{\dfrac{A \to B \vdash A \to B \qquad \dfrac{\vdash \neg\neg A \to A \qquad \neg\neg A \vdash \neg\neg A}{\neg\neg A \vdash A}\to_E}{A \to B, \neg\neg A \vdash B}\to_E}{A \to B \vdash \neg\neg A \to B}\to_I}{\dfrac{(\neg\neg A \to B) \to C, A \to B \vdash C}{(\neg\neg A \to B) \to C \vdash (A \to B) \to C}\to_I}\to_E$$

$$\frac{}{\Gamma, A \vdash A}\ axiom$$

$$\frac{\Gamma, A \vdash \mathcal{C}_\pi[B]}{\Gamma \vdash \mathcal{C}_\pi[A \to B]} \to_I (\pi)$$

$$\frac{\Gamma \vdash \mathcal{C}^1_{\pi_1}[A \to B] \qquad \Gamma \vdash \mathcal{C}^2_{\pi_2}[A]}{\Gamma \vdash \mathcal{C}^1_{\pi_1}[\mathcal{C}^2_{\pi_2}[B]]} \to_{\overrightarrow{E}} (\pi_1; \pi_2)$$

$$\frac{\Gamma \vdash \mathcal{C}^1_{\pi_1}[A \to B] \qquad \Gamma \vdash \mathcal{C}^2_{\pi_2}[A]}{\Gamma \vdash \mathcal{C}^2_{\pi_2}[\mathcal{C}^1_{\pi_1}[B]]} \to_{\overleftarrow{E}} (\pi_1; \pi_2)$$

Note: π, π_1 and π_2 must be positive positions.

Figure: The contextual natural deduction calculus **NDc**

Double negation elimination axiom schema:

$$dne : \neg\neg F \rightarrow F$$

Deriving $(A \rightarrow B) \rightarrow C$ from $(\neg\neg A \rightarrow B) \rightarrow C$ in **NDc**:

$$\dfrac{\vdash \neg\neg A \rightarrow A \qquad (\neg\neg A \rightarrow B) \rightarrow C \vdash (\neg\neg A \rightarrow B) \rightarrow C}{(\neg\neg A \rightarrow B) \rightarrow C \vdash (A \rightarrow B) \rightarrow C} \rightarrow_E^{\leftarrow} (\epsilon; 11)$$

And in **ND**:

$$\dfrac{\dfrac{(\neg\neg A \rightarrow B) \rightarrow C \vdash (\neg\neg A \rightarrow B) \rightarrow C \qquad \dfrac{\dfrac{A \rightarrow B \vdash A \rightarrow B \qquad \dfrac{\vdash \neg\neg A \rightarrow A \qquad \neg\neg A \vdash \neg\neg A}{\neg\neg A \vdash A} \rightarrow_E}{A \rightarrow B, \neg\neg A \vdash B} \rightarrow_E}{A \rightarrow B \vdash \neg\neg A \rightarrow B} \rightarrow_I}{(\neg\neg A \rightarrow B) \rightarrow C, A \rightarrow B \vdash C} \rightarrow_E}{(\neg\neg A \rightarrow B) \rightarrow C \vdash (A \rightarrow B) \rightarrow C} \rightarrow_I$$

Skolemization axiom schema:

$$sk : \exists x.F[x] \rightarrow F[f_{sk}(x_1, \ldots, x_n)]$$

where $x_1, \ldots, x_n$ free-variables of F and f_{sk} new skolem symbol.

Deriving skolemization $(A \rightarrow B) \rightarrow P(c)$ from $(A \rightarrow B) \rightarrow \exists x.P(x)$ in **NDc**:

$$\dfrac{\vdash \exists x.P(x) \rightarrow P(c) \qquad (A \rightarrow B) \rightarrow \exists x.P(x) \vdash (A \rightarrow B) \rightarrow \exists x.P(x)}{(A \rightarrow B) \rightarrow \exists x.P(x) \vdash (A \rightarrow B) \rightarrow P(c)} \rightarrow_E^{\leftarrow} (\epsilon; 0)$$

And in **ND**:

$$\dfrac{\dfrac{\vdash \exists x.P(x) \rightarrow P(c) \qquad \dfrac{\ldots \vdash A \rightarrow B \qquad \ldots \vdash (A \rightarrow B) \rightarrow \exists x.P(x)}{(A \rightarrow B) \rightarrow \exists x.P(x), A \rightarrow B \vdash \exists x.P(x)} \rightarrow_E}{A \rightarrow B, (A \rightarrow B) \rightarrow \exists x.P(x) \vdash P(c)} \rightarrow_E}{(A \rightarrow B) \rightarrow \exists x.P(x) \vdash \lambda c^{A \rightarrow B}.(A \rightarrow B) \rightarrow P(c)} \rightarrow_I$$

$$\frac{}{\Gamma, a : A \vdash a : A}\ axiom$$

$$\frac{\Gamma, a : A \vdash b : B}{\Gamma \vdash \lambda a^A . b : A \rightarrow B} \rightarrow_I$$

$$\frac{\Gamma \vdash f : A \rightarrow B \qquad \Gamma \vdash a : A}{\Gamma \vdash (f\ a) : B} \rightarrow_E$$

Figure: The natural deduction calculus **ND**

$$\frac{}{\Gamma, a : A \vdash a : A}\ axiom$$

$$\frac{\Gamma, a : A \vdash b : \mathcal{C}_\pi[B]}{\Gamma \vdash \lambda_\pi a^A.b : \mathcal{C}_\pi[A \rightarrow B]} \rightarrow_I (\pi)$$

$$\frac{\Gamma \vdash f : \mathcal{C}^1_{\pi_1}[A \rightarrow B] \qquad \Gamma \vdash a : \mathcal{C}^2_{\pi_2}[A]}{\Gamma \vdash (f\ a)^{\rightharpoonup}_{(\pi_1;\pi_2)} : \mathcal{C}^1_{\pi_1}[\mathcal{C}^2_{\pi_2}[B]]} \rightarrow^{\rightharpoonup}_E (\pi_1;\pi_2)$$

$$\frac{\Gamma \vdash f : \mathcal{C}^1_{\pi_1}[A \rightarrow B] \qquad \Gamma \vdash a : \mathcal{C}^2_{\pi_2}[A]}{\Gamma \vdash (f\ a)^{\leftharpoonup}_{(\pi_1;\pi_2)} : \mathcal{C}^2_{\pi_2}[\mathcal{C}^1_{\pi_1}[B]]} \rightarrow^{\leftharpoonup}_E (\pi_1;\pi_2)$$

Note: π, π_1 and π_2 must be positive positions.

Figure: The contextual natural deduction calculus **NDc**

Deriving $(A \to B) \to C$ from $(\neg\neg A \to B) \to C$ in **NDc**:

$$\dfrac{\vdash dne : \neg\neg A \to A \qquad a : \ldots \vdash a : (\neg\neg A \to B) \to C}{a : (\neg\neg A \to B) \to C \vdash (dne\ a)^{\leftarrow}_{(\epsilon;11)} : (A \to B) \to C} \to^{\leftarrow}_{E} (\epsilon; 11)$$

And in **ND**:

$$\dfrac{\dfrac{a : \ldots \vdash a : (\neg\neg A \to B) \to C \qquad \dfrac{\dfrac{c : \ldots \vdash c : A \to B \qquad \dfrac{\vdash dne : \neg\neg A \to A \qquad d : \ldots \vdash d : \neg\neg A}{d : \ldots \vdash (dne\ d) : A} \to}{d : \ldots, c : \ldots \vdash (c\ (dne\ d)) : B} \to_E}{c : \ldots \vdash \lambda d^{\neg\neg A}.(c\ (dne\ d)) : \neg\neg A \to B} \to_I}{a : \ldots, c : \ldots \vdash (a\ \lambda d.(c\ (dne\ d))) : C} \to_E}{a : (\neg\neg A \to B) \to C \vdash \lambda c^{A \to B}.(a\ \lambda d^{\neg\neg A}.(c\ (dne\ d))) : (A \to B) \to C} \to_I$$

Deriving skolemization $(A \to B) \to P(c)$ from $(A \to B) \to \exists x.P(x)$ in **NDc**:

$$\dfrac{\vdash sk : \exists x.P(x) \to P(c) \qquad a : \ldots \vdash a : (A \to B) \to \exists x.P(x)}{a : (A \to B) \to \exists x.P(x) \vdash (sk\ a)^{\leftarrow}_{(\epsilon;0)} : (A \to B) \to P(c)} \to^{\leftarrow}_{E} (\epsilon;0)$$

And in **ND**:

$$\dfrac{\dfrac{\vdash sk : \exists x.P(x) \to P(c) \qquad \dfrac{c : \ldots \vdash c : A \to B \qquad a : \ldots \vdash a : (A \to B) \to \exists x.P(x)}{a : (A \to B) \to \exists x.P(x), c : A \to B \vdash (a\ c) : \exists x.P(x)} \to_E}{c : A \to B, a : (A \to B) \to \exists x.P(x) \vdash (sk\ (a\ c)) : P(c)} \to_E}{a : (A \to B) \to \exists x.P(x) \vdash \lambda c^{A \to B}.(sk\ (a\ c)) : (A \to B) \to P(c)} \to_I$$

Completeness

Theorem (Completeness)

*If T is provable in **ND**, then T is provable in **NDc**.*

Definition (Translation of λ-terms into λ^d-terms)

- $\zeta[v] \doteq v$ (for a variable v).
- $\zeta[\lambda v^T.t] \doteq \lambda_\epsilon v^T.\zeta[t]$
- $\zeta[(m\ n)] \doteq (\zeta[m]\ \zeta[n])_{(\epsilon;\epsilon)}$

Soundness

Theorem (Soundness)

*If T is provable in **NDc**, then T is provable in **ND**.*

- If t is an $(\rightharpoonup)$-application of the form $(f\ a)^{\rightharpoonup}_{(\pi_1;\pi_2)}$, the translation is defined by two successive inductions, firstly on the position π_1 and then (when $\pi_1 = \epsilon$) on π_2, according to the cases below:
  - If $\pi_1 = 0\pi$, it is the case that t matches $(f^{C \rightarrow D}\ a)^{\rightharpoonup}_{(0\pi;\pi_2)}$, and then
  $$\xi[t] \doteq \lambda c^C.\xi[((f\ c)\ a))^{\rightharpoonup}_{(\pi;\pi_2)}]$$
  - If $\pi_1 = 1\pi'$, then there is at least one occurrence of the digit 1 in π', since π_1 is positive and π' is negative. Therefore, π_1 is necessarily of the form $10\ldots01\pi$ and t matches $(f^{(C_1 \rightarrow \ldots C_n \rightarrow (T_\pi[A \rightarrow B] \rightarrow D_1)) \rightarrow D_2}\ a)^{\rightharpoonup}_{(10\ldots01\pi;\pi_2)}$. Then
  $$\xi[t] \doteq \lambda k^{C_1 \rightarrow \ldots C_n \rightarrow (T_\pi[B] \rightarrow D_1)}.(f\ \lambda c_1^{C_1} \ldots c_n^{C_n}.\lambda h^{T_\pi[A \rightarrow B]}.(k\ c_1 \ldots c_n\ \xi[(h\ a)^{\rightharpoonup}_{(\pi;\pi_2)}])$$
  - If $\pi_1 = \epsilon$ and $\pi_2 = 0\pi$, it is the case that t matches $(f\ a^{C \rightarrow D})^{\rightharpoonup}_{(\epsilon;0\pi)}$, and then
  $$\xi[t] \doteq \lambda c^C.\xi[(f\ (a\ c))^{\rightharpoonup}_{(\epsilon;\pi)}]$$
  - If $\pi_1 = \epsilon$ and $\pi_2 = 1\pi'$, then there is at least one occurrence of the digit 1 in π', since π_2 is positive and π' is negative. Therefore, π_2 is of the form $10\ldots01\pi$ and t matches $(f^{A \rightarrow B}\ a^{(C_1 \rightarrow \ldots C_n \rightarrow (T_\pi[A] \rightarrow D_1)) \rightarrow D_2})^{\rightharpoonup}_{(\epsilon;10\ldots01\pi)}$. Then
  $$\xi[t] \doteq \lambda k^{C_1 \rightarrow \ldots C_n \rightarrow (T_\pi[B] \rightarrow D_1)}.(a\ \lambda c_1^{C_1} \ldots c_n^{C_n}.\lambda h^{T_\pi[A]}.(k\ c_1 \ldots c_n\ \xi[(f\ h)^{\rightharpoonup}_{(\epsilon;\pi)}]))$$
  - If $\pi_1 = \pi_2 = \epsilon$, it is the case that t matches $(f\ a)^{\rightharpoonup}_{(\epsilon;\epsilon)}$, and then
  $$\xi[t] \doteq (\xi[f]\ \xi[a])$$

- If t is an $(\leftharpoonup)$-application of the form $(f\ a)^{\leftharpoonup}_{(\pi_1;\pi_2)}$, the translation is analogous to the previous case for $(f\ a)^{\rightharpoonup}_{(\pi_1;\pi_2)}$, but the induction is made firstly on the position π_2 and only then (when $\pi_2 = \epsilon$) on π_1. For the sake of clarity, all cases are shown below:
  - If $\pi_2 = 0\pi$, it is the case that t matches $(f\ a^{C\rightarrow D})^{\leftharpoonup}_{(\pi_1;0\pi)}$, and then
    $$\xi[t] \doteq \lambda c^C.\xi[(f\ (a\ c))^{\leftharpoonup}_{(\pi_1;\pi)}]$$
  - If $\pi_2 = 1\pi'$, then there is at least one occurrence of the digit 1 in π', since π_2 is positive and π' is negative. Therefore, π_2 is necessarily of the form $10\ldots01\pi$ and t matches $(f\ a^{(C_1\rightarrow\ldots C_n\rightarrow(T_\pi[A]\rightarrow D_1))\rightarrow D_2})^{\leftharpoonup}_{(\pi_1;10\ldots01\pi)}$. Then
    $$\xi[t] \doteq \lambda k^{C_1\rightarrow\ldots C_n\rightarrow(T_\pi[B]\rightarrow D_1)}.(a\ \lambda c_1^{C_1}\ldots c_n^{C_n}.\lambda h^{T_\pi[A]}.(k\ c_1\ldots c_n\ \xi[(f\ h)^{\leftharpoonup}_{(\pi_1;\pi)}]))$$
  - If $\pi_2 = \epsilon$ and $\pi_1 = 0\pi$, it is the case that t matches $(f^{C\rightarrow D}\ a)^{\leftharpoonup}_{(0\pi;\epsilon)}$, and then
    $$\xi[t] \doteq \lambda c^C.\xi[((f\ c)\ a)^{\leftharpoonup}_{(\pi;\epsilon)}]$$
  - If $\pi_2 = \epsilon$ and $\pi_1 = 1\pi'$, then there is at least one occurrence of the digit 1 in π', since π_1 is positive and π' is negative. Consequently, π_1 is of the form $10\ldots01\pi$ and t matches $(f^{(C_1\rightarrow\ldots C_n\rightarrow(T_\pi[A\rightarrow B]\rightarrow D_1))\rightarrow D_2}\ a)^{\leftharpoonup}_{(10\ldots01\pi;\epsilon)}$. Then
    $$\xi[t] \doteq \lambda k^{C_1\rightarrow\ldots C_n\rightarrow(T_\pi[B]\rightarrow D_1)}.(f\ \lambda c_1^{C_1}\ldots c_n^{C_n}.\lambda h^{T_\pi[A\rightarrow B]}.(k\ c_1\ldots c_n\ \xi[(h\ a)^{\leftharpoonup}_{(\pi;\epsilon)}])$$
  - If $\pi_2 = \pi_1 = \epsilon$, it is the case that t matches $(f\ a)^{\leftharpoonup}_{(\epsilon;\epsilon)}$, and then
    $$\xi[t] \doteq (\xi[f]\ \xi[a])$$

- If t is a variable, then $\xi[t] \doteq t$
- If t is an abstraction of the form $\lambda_\pi a^A.b$, the translation is defined by induction on the position π, according to the cases below:
  - If $\pi = 0\pi'$, it is the case that t matches $\lambda_{0\pi'} a^A.b^{C\to D}$, and then
    $$\xi[t] \doteq \lambda c^C.\xi[\lambda_{\pi'} a^A.(bc)]$$
  - If $\pi = 1\pi'$, then there is at least one occurrence of the digit 1 in π', since π is positive and π' is negative. Therefore, π is necessarily of the form $10\ldots01\pi''$ and t matches $\lambda_{10\ldots01\pi''} a^A.f^{(C_1\to\ldots C_n\to(T_{\pi''}[B]\to D_1))\to D_2}$. Then
    $$\xi[t] \doteq \lambda k^{C_1\to\ldots C_n\to(T_{\pi''}[A\to B]\to D_1)}.(f\ \lambda c_1^{C_1}\ldots c_n^{C_n}.\lambda h^{T_{\pi''}[B]}.(k\ c_1\ldots c_n\ \xi[\lambda_{\pi''} a^A.h])$$
  - If $\pi = \epsilon$, it is the case that t matches $\lambda_\epsilon a.f$, and then
    $$\xi[t] \doteq \lambda a.\xi[f]$$

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    $$\xi[t] \doteq \lambda k^{C_1 \rightarrow \ldots C_n \rightarrow (T_{\pi''}[A \rightarrow B] \rightarrow D_1)}.(f\ \lambda c_1^{C_1} \ldots c_n^{C_n}.\lambda h^{T_{\pi''}[B]}.(k\ c_1 \ldots c_n\ \xi[\lambda_{\pi''} a^A.h])$$

  - If $\pi = \epsilon$, it is the case that t matches $\lambda_\epsilon a.f$, and then

    $$\xi[t] \doteq \lambda a.\xi[f]$$

Intuitionistic Contextual Soundness Condition:
If π contains the digit 1, then a is not allowed to occur in f.

$$\dfrac{\dfrac{\overline{p : P, a : (Q \to P) \vdash p : P}\ axiom}{p : P \vdash \lambda a^{(Q \to P)}.p : (Q \to P) \to P} \to_I}{\vdash \lambda_{11} p^{P}.\lambda a^{(Q \to P)}.p : ((P \to Q) \to P) \to P} \to_I (11)$$

$$\dfrac{\dfrac{\overline{p : P, a : (\bot \to \bot) \vdash p : P}\ axiom}{p : P \vdash \lambda a^{(\bot \to \bot)}.p : (\bot \to \bot) \to P} \to_I}{\vdash \lambda_{11} p^{P}.\lambda a^{(\bot \to \bot)}.p : ((P \to \bot) \to \bot) \to P} \to_I (11)$$

- Add classical principles as axioms to shallow natural deduction
- Use a multi-conclusion natural deduction calculus
- Allow unrestricted contextual natural deduction inference rules

Normalization

$$(\lambda a^A.f\ t') \rightsquigarrow_\beta f[a \backslash t']$$

$$(\lambda_0 a^A.f\ t')_{(0;\epsilon)} \rightsquigarrow^? f[a \backslash t']$$

$$(\lambda b^B.\lambda a^A.(f\ b)\ t')_{(0;\epsilon)} \rightsquigarrow^? \lambda b^B.(f[a \backslash t']\ b)$$

$$\frac{\lambda_{0\pi} a^A . b^{C \to D}}{\lambda c^C . \lambda_\pi a^A . (bc)}$$

$$\frac{\lambda_{10 \ldots 01\pi} a^A . f^{(C_1 \to \ldots C_n \to (T_\pi[B] \to D_1)) \to D_2}}{\lambda k^{C_1 \to \ldots C_n \to (T_\pi[A \to B] \to D_1)} . (f \ \lambda c_1^{C_1} \ldots c_n^{C_n} . \lambda h^{T_\pi[B]} . (k \ c_1 \ldots c_n \ \lambda_\pi a^A . h)}$$

Figure: Unfolding Contextual Abstractions

$$\frac{(f^{C \to D} \ a)^{\rightharpoonup}_{(0\pi;\pi_2)}}{\lambda c^C . ((f \ c) \ a))^{\rightharpoonup}_{(\pi;\pi_2)}} \qquad \frac{(f \ a^{C \to D})^{\leftharpoonup}_{(\pi_1;0\pi)}}{\lambda c^C . (f \ (a \ c))^{\leftharpoonup}_{(\pi_1;\pi)}}$$

$$\frac{(f \ a^{C \to D})^{\rightharpoonup}_{(\epsilon;0\pi)}}{\lambda c^C . (f \ (a \ c))^{\rightharpoonup}_{(\epsilon;\pi)}} \qquad \frac{(f^{C \to D} \ a)^{\leftharpoonup}_{(0\pi;\epsilon)}}{\lambda c^C . ((f \ c) \ a)^{\leftharpoonup}_{(\pi;\epsilon)}}$$

Figure: Unfolding Contextual Applications with Position Starting with 0

Unfolding

$$\frac{(f^{(C_1\to\ldots C_n\to(T_\pi[A\to B]\to D_1))\to D_2}\ a)^{\rightharpoonup}_{(10\ldots01\pi;\pi_2)}}{\lambda k^{C_1\to\ldots C_n\to(T_\pi[B]\to D_1)}.(f\ \lambda c_1^{C_1}\ldots c_n^{C_n}.\lambda h^{T_\pi[A\to B]}.(k\ c_1\ldots c_n\ (h\ a)^{\rightharpoonup}_{(\pi;\pi_2)})}$$

$$\frac{(f\ a^{(C_1\to\ldots C_n\to(T_\pi[A]\to D_1))\to D_2})^{\leftharpoonup}_{(\pi_1;10\ldots01\pi)}}{\lambda k^{C_1\to\ldots C_n\to(T_\pi[B]\to D_1)}.(a\ \lambda c_1^{C_1}\ldots c_n^{C_n}.\lambda h^{T_\pi[A]}.(k\ c_1\ldots c_n\ (f\ h)^{\leftharpoonup}_{(\pi_1;\pi)}))}$$

$$\frac{(f^{A\to B}\ a^{(C_1\to\ldots C_n\to(T_\pi[A]\to D_1))\to D_2})^{\rightharpoonup}_{(\epsilon;10\ldots01\pi)}}{\lambda k^{C_1\to\ldots C_n\to(T_\pi[B]\to D_1)}.(a\ \lambda c_1^{C_1}\ldots c_n^{C_n}.\lambda h^{T_\pi[A]}.(k\ c_1\ldots c_n\ (f\ h)^{\rightharpoonup}_{(\epsilon;\pi)}))}$$

$$\frac{(f^{(C_1\to\ldots C_n\to(T_\pi[A\to B]\to D_1))\to D_2}\ a)^{\leftharpoonup}_{(10\ldots01\pi;\epsilon)}}{\lambda k^{C_1\to\ldots C_n\to(T_\pi[B]\to D_1)}.(f\ \lambda c_1^{C_1}\ldots c_n^{C_n}.\lambda h^{T_\pi[A\to B]}.(k\ c_1\ldots c_n\ (h\ a)^{\leftharpoonup}_{(\pi;\epsilon)})}$$

Figure: Unfolding Contextual Applications with Position Starting with 1

$$
\begin{aligned}
(\lambda_0 a^A.(\lambda_0 b^B.h\ a)\ t')_{(0;\epsilon)} &\rightsquigarrow_\delta && (\lambda b_1^B.\lambda a^A.((\lambda_0 b^B.h\ a)\ b_1)\ t')_{(0;\epsilon)} \\
&\rightsquigarrow_\delta && \lambda b_2^B.((\lambda b_1^B.\lambda a^A.((\lambda_0 b^B.h\ a)\ b_1)\ b_2)\ t') \\
&\rightsquigarrow_\beta && \lambda b_2^B.(\lambda a^A.((\lambda_0 b^B.h\ a)\ b_2)\ t') \\
&\rightsquigarrow_\beta && \lambda b_2^B.((\lambda_0 b^B.h\ t')\ b_2) \\
&=_\eta && (\lambda_0 b^B.h\ t')
\end{aligned}
$$

- $\rightsquigarrow_\delta$ is terminating.
  - For all unfolding rules, the sum of the sizes of all positions decreases.
- $\rightsquigarrow_\delta$ is locally confluent.
  - There are no critical pairs.
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Beta-Unfolding

- $\rightsquigarrow_{\beta\delta}$ is weakly normalizing.
  - Just unfold first and beta-reduce later.
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In the best cases,
NDc-proofs can be quadratically smaller than smallest **ND**-proofs.

There is a sequence of theorems F_n whose smallest **ND**-proofs ψ_n grow at least quadratically (i.e. $s(\psi_n) \in \Omega(n^2)$), while there are **NDc**-proofs ψ_n^d of F_n growing at most linearly (i.e. $s(\psi_n^d) \in O(n)$).

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while there are
NDc-proofs ψ_n^d of F_n growing at most linearly (i.e. $s(\psi_n^d) \in O(n)$).

Definition (Size of a Type)

- $s(A) \doteq 1$ (if A is an atomic type)
- $s(T_1 \rightarrow T_2) \doteq 1 + s(T_1) + s(T_2)$

Definition (Size of a λ-term)

- $s(v) \doteq 1$ (if v is a variable)
- $s(\lambda v^T.t') \doteq 2 + s(T) + s(t')$
- $s((m\ n)) \doteq 1 + s(m) + s(n)$

Definition (Size of a λ^d-term)

- $s(v) \doteq 1$ (if v is a variable)
- $s(\lambda_\pi v^T.t') \doteq 2 + s(T) + s(t') + s(\pi)$
- $s((m\ n)^{\rightharpoonup}_{\pi_1;\pi_2}) \doteq 1 + s(m) + s(n) + s(\pi_1) + s(\pi_2)$
- $s((m\ n)^{\leftharpoonup}_{\pi_1;\pi_2}) \doteq 1 + s(m) + s(n) + s(\pi_1) + s(\pi_2)$

Let $F_n \doteq T^n(A \to B) \to (A \to T^n(B))$ where:

$$T^0(F) \doteq F$$
$$T^n(F) \doteq (T^{n-1}(F) \to D_{2n-1}) \to D_{2n}$$

Let $\psi_n^d \doteq \mathcal{I}_d^{-1}(t_n^d)$ where:

$$t_n^d \doteq \lambda f^{T^n(A \to B)}.\lambda a^A.(f\ a)_{(\underbrace{11\ldots1}_{2n};\epsilon)}$$

Let $\psi_n \doteq \mathcal{I}^{-1}(t_n)$ where:

$$t_n \doteq \xi(t_n^d)$$

Note that ψ_k is a smallest **ND**-proof of F_k. Any **ND**-proof of F_k must (at least) decompose F_k until the subformulas $A \to B$ and A are obtained and then apply $A \to B$ to A. ψ_k does exactly this and nothing more.

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By definition, $s(\psi_n^d) = s(t_n^d)$, and $s(t_n^d)$ is computed below:

$$\begin{aligned}
s(t_n^d) &= s(\lambda f^{T^n(A\to B)}.\lambda a^A.(f\ a)_{(\underbrace{11\ldots1}_{2n};\epsilon)}) \\
&= 2 + s(T^n(A \to B)) + s(\lambda a^A.(f\ a)_{(11\ldots1;\epsilon)}) \\
&= 2 + (3 + 4n) + s(\lambda a^A.(f\ a)_{(11\ldots1;\epsilon)}) \\
&= 5 + 4n + (2 + s(A) + s((f\ a)_{(11\ldots1;\epsilon)})) \\
&= 8 + 4n + s((f\ a)_{(11\ldots1;\epsilon)}) \\
&= 8 + 4n + (1 + s(f) + s(a) + s(\underbrace{11\ldots1}_{2n}) + s(\epsilon)) \\
&= 8 + 4n + (3 + 2n + 0) \\
&= 11 + 6n
\end{aligned}$$

Therefore, $s(t_n^d) \in O(n)$.

By definition, $s(\psi_n) = s(t_n)$, and $s(t_n)$ is computed below:

$$\begin{aligned} s(t_n) &= s(\xi(\lambda f^{T^n(A\to B)}.\lambda a^A.(f\ a)_{(\underbrace{11\ldots1}_{2n};\epsilon)})) \\ &= s(\lambda f^{T^n(A\to B)}.\lambda a^A.\xi((f\ a)_{(11\ldots1;\epsilon)})) \\ &= 2 + s(T^n(A\to B)) + 2 + s(A) + s(\xi((f\ a)_{(11\ldots1;\epsilon)})) \\ &= 8 + 4n + s(\xi((f\ a)_{(\underbrace{11\ldots1}_{2n};\epsilon)})) \end{aligned}$$

$s(t_n) = 8 + 4n + s(\xi(((f\ a)_{(\underbrace{11\ldots1}_{2n};\epsilon)}))$. Let $q(n) \doteq s(\xi(((f\ a)_{(\underbrace{11\ldots1}_{2n};\epsilon)}))$.

Then:

$$q(0) = s(\xi(((f\ a)_{(\epsilon;\epsilon)})) = 3$$

$$\begin{aligned}
q(n) &= s(\xi(((f\ a)_{(\underbrace{1111\ldots1}_{2n};\epsilon)})) \\
&= s(\lambda_k^{T^{n-1}(B)\to D_{2n-1}}.(f\ \lambda h^{T^{n-1}(A\to B)}.\xi((h\ a)_{(\underbrace{11\ldots1}_{2n-2};\epsilon)}))) \\
&= 2 + s(T^{n-1}(B) \to D_{2n-1}) + s((f\ \lambda h^{T^{n-1}(A\to B)}.\xi((h\ a)_{(11\ldots1;\epsilon)}))) \\
&= 2 + 4(n-1) + 3 + s((f\ \lambda h^{T^{n-1}(A\to B)}.\xi((h\ a)_{(11\ldots1;\epsilon)}))) \\
&= 1 + 4n + s((f\ \lambda h^{T^{n-1}(A\to B)}.\xi((h\ a)_{(11\ldots1;\epsilon)}))) \\
&= 1 + 4n + 2 + s(\lambda h^{T^{n-1}(A\to B)}.\xi((h\ a)_{(11\ldots1;\epsilon)})) \\
&= 3 + 4n + s(\lambda h^{T^{n-1}(A\to B)}.\xi((h\ a)_{(11\ldots1;\epsilon)})) \\
&= 5 + 4n + s(T^{n-1}(A \to B)) + s(\xi((h\ a)_{(\underbrace{11\ldots1}_{2n-2};\epsilon)})) = 4 + 8n + q(n-1)
\end{aligned}$$

$$s(t_n) = 8 + 4n + q(n)$$

$$q(0) = 3$$

$$q(n) = 4 + 8n + q(n-1)$$

Solving the recurrence relation above gives the following closed-form for q:

$$q(n) = 4n^2 + 8n + 3$$

Therefore, $s(\psi_n) \in \Omega(n^2)$.

Proof Compression by Folding

- $\rightsquigarrow_{\delta}^{-1}$ is terminating
  - The term size decreases with every inverse rewriting step.
- $\rightsquigarrow_{\delta}^{-1}$ is not confluent
  - Let $f : A \rightarrow B$ and $a : (A \rightarrow D) \rightarrow E$. Then:
    - $\lambda k^{B\rightarrow D}.(a\ \lambda h^{A}.(k\ (f\ h))) \quad \rightsquigarrow_{\delta}^{-1} \quad (f\ a)_{(\epsilon;11)}$
    - $\lambda k^{B\rightarrow D}.(a\ \lambda h^{A}.(k\ (f\ h))) \quad \rightsquigarrow_{\delta}^{-1} \quad \lambda k^{B\rightarrow D}.(a\ (k\ f)_{(\epsilon;0)})$

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$$h_{ab} : A \to B \qquad h_{bc} : B \to C \qquad h_{ade} : (A \to D) \to E$$

$$\lambda h_{cd}^{C \to D}.(h_{ade}\ \lambda h_{a}^{A}.(h_{cd}\ (h_{bc}\ (h_{ab}\ h_{a})))) : (C \to D) \to E$$

Normal form w.r.t. $\rightsquigarrow_{\delta}^{-1}$.
Yet, there are smaller λ^{d}-terms:

$$(h_{bc}\ (h_{ab}\ h_{ade})_{(\epsilon;11)})_{(\epsilon;11)}$$

$$((h_{bc}\ h_{ab})_{(\epsilon;0)}\ h_{ade})_{(\epsilon;11)}$$

To obtain them, we need folding + beta expansion

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Propositional Resolution

Current Work

- Contextual Natural Deduction:
  - go beyond the implicational fragment
  - investigate beta-expansion / cut-introduction
  - implement and evaluate compressibility in practice
  - obtain a syntactic proof of soundness for the classical case
  - investigate algorithmic interpretations for the classical case
- Propositional Resolution:
  - develop efficient subsumption algorithms
  - improve lowering of subproofs
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The End

- Thanks!
- Some announcements:
  - LowerUnivalents: SMT2013, Helsinki, 8th of July 15:30
  - Proof Compression Workshop:
    16th of September, affiliated with Tableaux, Nancy, France
- Questions? Comments? Suggestions?
- www.logic.at/people/bruno/