Categorical Semantics for Functional Reactive Programming with Temporal Recursion and Corecursion

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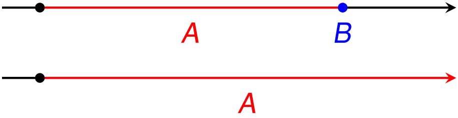
Functional Reactive Programming (FRP)

- programming paradigm for treating temporal aspects in a declarative fashion
- two key features:
  - time-dependent type membership
  - temporal type constructors
- Curry–Howard correspondence to temporal logic:
  - time-dependent trueness
  - temporal operators
- time:
  - linear
  - not necessarily discrete

Processes

- process consists of a continuous part and optionally a terminal event:

$A \rhd'' B$:



- different process types with different termination guarantees:
  - nontermination possible
  - termination guaranteed
  - termination guaranteed with upper bound on termination time

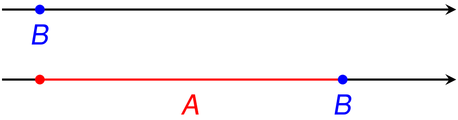
Processes that deal with the present

- processes that start immediately:

$A \rhd' B$:

- processes that may terminate immediately:

$A \rhd B$:



- $\rhd'$ and $\rhd$ definable in terms of $\rhd''$:

$$A \rhd' B = A \times A \rhd'' B \qquad A \rhd B = B + A \rhd' B$$

Abstract process categories (APCs)

- cartesian closed category $\mathcal{C}$ with coproducts
- functors that model process type constructors:

$$\triangleright'' : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$$

- natural transformations that model FRP operations:
  - ideal monads
  - ideal comonads
  - further structure (not in this talk)

Ideal monads

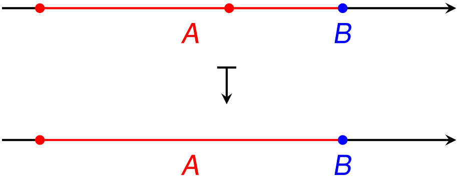
- each $A \rhd' -$ is an ideal monad:

$$\mu'_B : A \rhd' (A \rhd B) \to A \rhd' B$$

- concatenation of a continuous part with a follow-up process:

$A \rhd' (A \rhd B)$:

$A \rhd' B$:



Ideal comonads

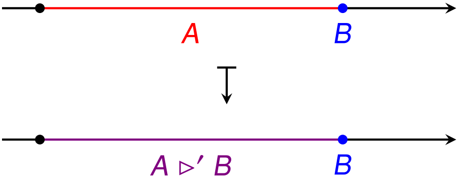
- each $- \rhd'' B$ is an ideal comonad:

$$\delta'_A : A \rhd'' B \to (A \rhd' B) \rhd'' B$$

- generation of a continuous part of shorter and shorter suffixes:

$A \rhd'' B$:

$(A \rhd' B) \rhd'' B$:

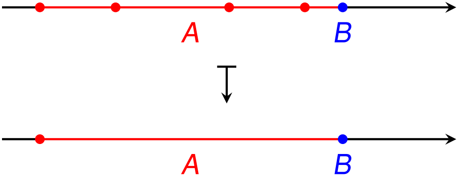


Iteration of ideal multiplication and comultiplication

- iterated concatenation via induction:

$\mu C\ .\ A \rhd' (B + C)$:

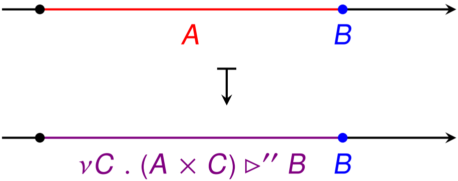
$A \rhd' B$:



- iterated suffix generation via coinduction:

$A \rhd'' B$:

$\nu C\ .\ (A \times C) \rhd'' B$:

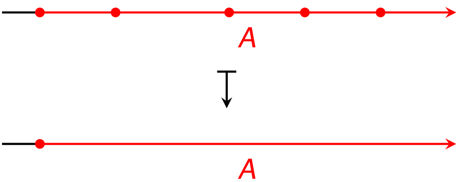


Wanted: Stronger variants of these iterations

- sequence of continuous parts may be infinite:

$\nu C \,.\, A \rhd' (B + C)$:

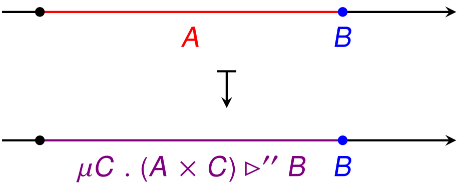
$A \rhd' B$:



- nesting depth must be finite:

$A \rhd'' B$:

$\mu C \,.\, (A \times C) \rhd'' B$:



Solution: Extending the ideal monad and comonad structure

- each $A \rhd' -$ is a completely iterative monad:

$$\frac{f : C \to A \rhd' (B + C)}{f^\infty : C \to A \rhd' B}$$

- each $- \rhd'' B$ is a recursive comonad:

$$\frac{f : (A \times C) \rhd'' B \to C}{f^* : A \rhd'' B \to C}$$

Are these extensions reasonable?

- check whether there are nontrivial instances of APCs that have the additional structure
- concrete process categories (CPCs) are instances of APCs
- do they have the required additional structure?

Concrete process categories

- make times explicit:
  - time scale can be any totally ordered set
- express causality of operations:
  - the prefix of a result that ends at a time t can only depend on the prefix of the argument that ends at t
  - operations expressed as families of prefix transformations, one for each t
- process types with simple termination guarantee cannot be modeled:
  - termination is a liveness property
  - only safety properties can be expressed, because only prefixes are considered
- the following process types can be modeled:

$\rhd_\infty$ nontermination possible

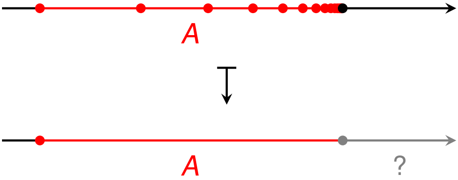
$\rhd_{t_b}$ termination at or before t_b guaranteed

A constraint on time scales

- infinitely many concatenations can be problematic:

$\nu C \,.\, A \rhd' (B + C)$:

$A \rhd' B$:



- analogous problem for suffix generation
- solution is to disallow “pathological” time scales:
  - every ascending sequence of times must be unbounded
  - Achilles catches up with the Tortoise
  - certain “interesting” time scales still allowed:

$$\{\, z + 1/n \mid z \in \mathbb{Z} \wedge n \in \mathbb{N} \setminus \{0\} \,\}$$

Compatibility with different termination (non)guarantees

- completely iterative monad:
  - $\triangleright_\infty$ infinitely many concatenations are no problem
  - $\triangleright_{t_b}$ only finitely many concatenations can occur, since all subprocesses terminate at or before t_b
- recursive comonad:
  - $\triangleright_{t_b}$ nesting depth is finite, since given process terminates
  - $\triangleright_\infty$ nesting depth is finite, since only finite prefixes of processes are considered

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