An Abstract Categorical Semantics for Functional Reactive Programming with Processes

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Functional Reactive Programming (FRP)

- programming paradigm for treating temporal aspects in a declarative fashion
- two key features:
  - time-dependent type membership
  - temporal type constructors
- Curry–Howard correspondence to temporal logic:
  - time-dependent trueness
  - temporal operators
- time:
  - linear
  - not necessarily discrete

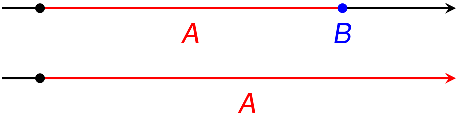
Processes

- process normally consists of a continuous part and a terminal event:

$A \rhd'' B$:

- weak process types allow for nonterminating processes:

$A \blacktriangleright'' B$:



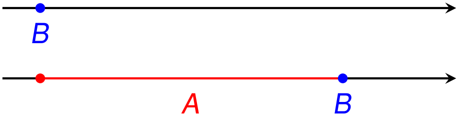
Processes that deal with the present

- processes that start immediately:

$A \rhd' B$:

- processes that may terminate immediately:

$A \rhd B$:



- $\rhd'$ and $\rhd$ definable in terms of $\rhd''$:

$$A \rhd' B = A \times A \rhd'' B \qquad A \rhd B = B + A \rhd' B$$

- weak variants $\blacktriangleright'$ and $\blacktriangleright$ can be defined analogously

Topic of this talk

- axiomatically defined categorical semantics for FRP with processes
- road to this semantics:
  1. categorical models of intuitionistic S4
     (Kobayashi 1997; Bierman and de Paiva 2000)
  2. temporal categories for FRP with behaviors and events
     (Jeltsch 2012)
  3. abstract process categories for FRP with processes
     (this talk)

Basic structure

- cartesian closed category $\mathcal{C}$ with coproducts
- functors that model strong and weak process type constructors:

$$\triangleright'', \blacktriangleright'' : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$$

- unified functor for modeling process type constructors:

$$- \triangleright''_{-} - : \mathbf{2} \times \mathcal{C} \times \mathcal{C} \to \mathcal{C}$$

- traditional functors by specialization:

$$\begin{aligned} \triangleright'' &= \triangleright''_0 \\ \blacktriangleright'' &= \triangleright''_1 \end{aligned}$$

- weakening as mapping:

$$\frac{w : 0 \to 1}{\triangleright''_w : \triangleright'' \to \blacktriangleright''}$$

Process expansion

- generates a continuous part of shorter and shorter suffixes
- three kinds of structures:
  - comonads:

$$\pi_1 : A \rhd'_W B \to A$$

$$\theta'_{A,W,B} : A \rhd'_W B \to \left(A \rhd'_W B\right) \rhd'_W B$$

  - ideal comonads:

$$\theta''_{A,W,B} : A \rhd''_W B \to \left(A \rhd'_W B\right) \rhd''_W B$$

  - “real comonads:”

$$\theta_{A,W,B} : A \rhd_W B \to \left(A \rhd'_W B\right) \rhd_W B$$

- derivation:

ideal comonads → comonads → “real comonads”

Process joining

- concatenates a continuous part with a follow-up process
- three kinds of structures:
  - monads:

$$\iota_1 : B \to A \rhd_W B$$
$$\vartheta_{A,W,B} : A \rhd_W (A \rhd_W B) \to A \rhd_W B$$

  - ideal monads:

$$\vartheta'_{A,W,B} : A \rhd'_W (A \rhd_W B) \to A \rhd'_W B$$

  - "gorgeous monads:"

$$\vartheta''_{A,W,B} : A \rhd''_W (A \rhd_W B) \to A \rhd''_W B$$

- derivation:

"gorgeous monads" → ideal monads → monads

Process merging and the canonical nonterminating process

- merging of two processes:

$$A_1 \rhd''_{W_1} B_1 \times A_2 \rhd''_{W_2} B_2$$
$$\downarrow$$
$$(A_1 \times A_2) \rhd''_{W_1 \times W_2} ((A_1, W_1, B_1) \odot (A_2, W_2, B_2))$$

- definition of $\odot$:

$$(A_1, W_1, B_1) \odot (A_2, W_2, B_2)$$
$$=$$
$$\left(B_1 \times B_2\right) + \left(B_1 \times A_2 \rhd'_{W_2} B_2\right) + \left(A_1 \rhd'_{W_1} B_1 \times B_2\right)$$

- $W_1 \times W_2$ is minimum of W_1 and W_2
- nullary version constructs the canonical nonterminating process:

$$1 \rightarrow 1 \rhd''_1 0$$

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Conclusions and further work

- developed abstract process categories (APCs):
  - categorical semantics for FRP with processes
  - axiomatically defined
  - generalize temporal categories (Jeltsch 2012)
- further work:
  - recursion and corecursion on processes
  - mutable state in FRP
  - FRP implementation with API inspired by (extended versions of) APCs

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