Abstract Categorical Semantics for Functional Reactive Programming with Resources

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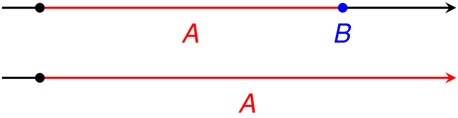
Functional reactive programming (FRP)

- programming paradigm for dealing with temporal aspects in a declarative fashion
- two key features:
  - time-dependent type membership
  - temporal type constructors
- Curry–Howard correspondence to temporal logic:
  - time-dependent trueness
  - temporal operators
- time:
  - linear
  - not necessarily discrete

Processes

- process consists of a continuous part and optionally a terminal event:

$A \rhd'' B$:



- different process types with different termination guarantees:
  - nontermination possible
  - termination guaranteed
  - termination guaranteed with upper bound on termination time

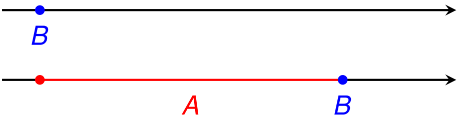
Processes that deal with the present

- processes that start immediately:

$A \rhd' B$:

- processes that may terminate immediately:

$A \rhd B$:



- $\rhd'$ and $\rhd$ definable in terms of $\rhd''$:

$$A \rhd' B = A \times A \rhd'' B \qquad A \rhd B = B + A \rhd' B$$

Functional programming with resources

- programming paradigm for dealing with resources in a declarative fashion
- examples of resources:
  - files
  - GUI widgets
  - threads in a concurrent program
- two key features:
  - resource-dependent type membership
  - resource-related type constructors
- Curry–Howard correspondence to the logic of bunched implications (which is similar to linear logic)

Primitive types

- correspond to resource types
- value of such a type describes how to construct a resource from the current resource
- examples:

File	file construction
Widget	widget construction
Thread	thread construction

Resource-related type constructors

- type constructors:
  - $\otimes$ resource splitting and simultaneous resource construction
  - I construction of the empty resource (destruction)
  - $\multimap$ consumption of additional resource
- structure of a symmetric monoidal closed category:
  - $\otimes$ is associative and commutative
  - I is neutral element of $\otimes$
  - $\multimap$ allows for Currying with respect to $\otimes$
- generally no duplication and disposal:

$$A \nrightarrow A \otimes A$$

$$A \nrightarrow I$$

Integration of FRP and FP with resources

- take constructs from both paradigms:

  processes $\rhd''$

  resources $\otimes$, I, $\multimap$

- extend processes:

  additionally describe resource transformation over time

- generalize semantics for FRP to become semantics for FRP with resources

Categorical semantics for FRP with and without resources

- basic structure:

  without resources: cartesian closed category $\mathcal{C}$ with coproducts ($\times$, 1, $\to$, $+$, and 0)

  with resources: cartesian closed category $\mathcal{C}$ with coproducts and a symmetric monoidal closed category structure (all of the above plus $\otimes$, I, and $\multimap$)

- functor that models process type constructor:

$$\triangleright'' : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$$

- natural transformations that model FRP operations

Process joining

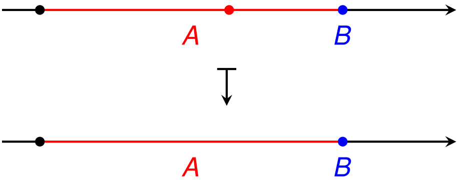
- each $A \rhd''$ – is something similar to an ideal monad:

$$\vartheta''_B : A \rhd'' (A \rhd B) \to A \rhd'' B$$

- concatenation of a continuous part with a follow-up process:

$A \rhd'' (A \rhd B)$:

$A \rhd'' B$:



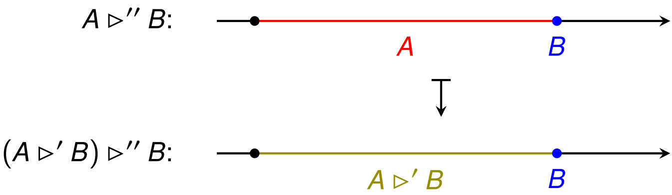
- makes sense with and without resource transformations in processes

Process expansion

- each $- \rhd'' B$ is an ideal comonad:

$$\theta''_A : A \rhd'' B \to (A \rhd' B) \rhd'' B$$

- generation of a continuous part of shorter and shorter suffixes:



- makes sense with and without resource transformations in processes

Process merging

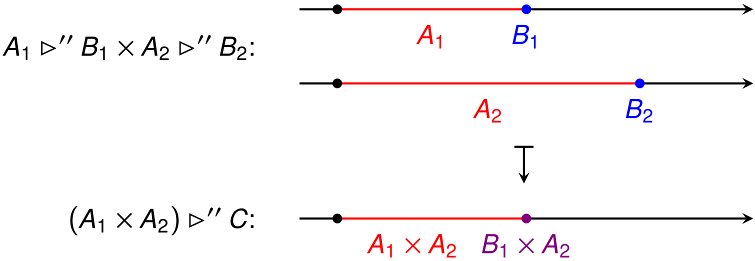
- binary coherence map of a lax symmetric monoidal functor:

$$A_1 \rhd'' B_1 \times A_2 \rhd'' B_2 \to (A_1 \times A_2) \rhd'' C$$

  with

$$C = B_1 \times B_2 + B_1 \times A_2 + A_1 \times B_2$$

- merging of two processes:



- for FRP with resources replace $\times$ by $\otimes$

Process merging

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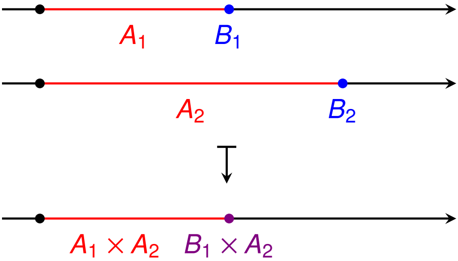

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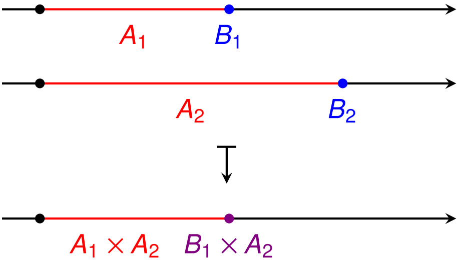
with

$$C = B_1 \times B_2 + {\color{red} B_1 \times A_2} + A_1 \times B_2$$

- merging of two processes:

$A_1 \rhd'' B_1 \times A_2 \rhd'' B_2$:

$(A_1 \times A_2) \rhd'' C$:



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Process merging

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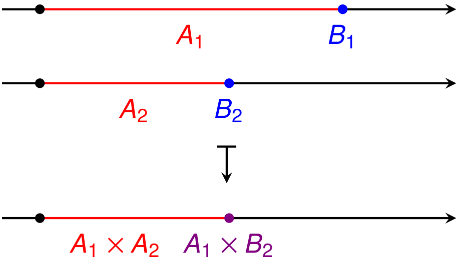
  with

$$C = B_1 \times B_2 + B_1 \times A_2 + {\color{red}A_1 \times B_2}$$

- merging of two processes:

$A_1 \rhd'' B_1 \times A_2 \rhd'' B_2$:

$(A_1 \times A_2) \rhd'' C$:



- for FRP with resources replace $\times$ by $\otimes$

Process merging

- binary coherence map of a lax symmetric monoidal functor:

$$A_1 \rhd'' B_1 \times A_2 \rhd'' B_2 \to (A_1 \times A_2) \rhd'' C$$

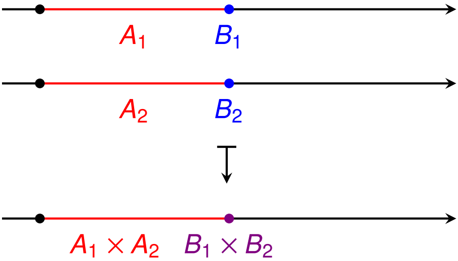
  with

$$C = \color{red}{B_1 \times B_2} + B_1 \times A_2 + A_1 \times B_2$$

- merging of two processes:

$A_1 \rhd'' B_1 \times A_2 \rhd'' B_2$:

$(A_1 \times A_2) \rhd'' C$:



- for FRP with resources replace $\times$ by $\otimes$

Process merging

- binary coherence map of a lax symmetric monoidal functor:

$$A_1 \rhd'' B_1 \times A_2 \rhd'' B_2 \to (A_1 \times A_2) \rhd'' C$$

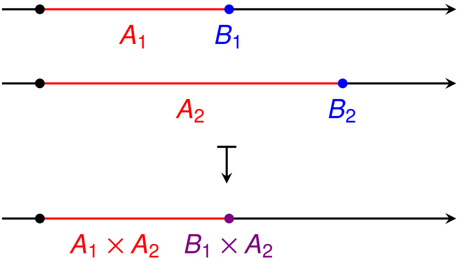
with

$$C = B_1 \times B_2 + B_1 \times A_2 + A_1 \times B_2$$

- merging of two processes:

$A_1 \rhd'' B_1 \times A_2 \rhd'' B_2$:

$(A_1 \times A_2) \rhd'' C$:



- for FRP with resources replace $\times$ by $\otimes$

Process merging

- binary coherence map of a lax symmetric monoidal functor:

$$A_1 \rhd'' B_1 \otimes A_2 \rhd'' B_2 \to (A_1 \otimes A_2) \rhd'' C$$

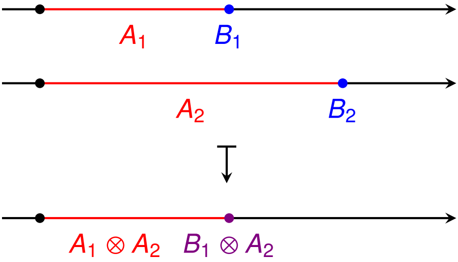
with

$$C = B_1 \otimes B_2 + B_1 \otimes A_2 + A_1 \otimes B_2$$

- merging of two processes:

$A_1 \rhd'' B_1 \otimes A_2 \rhd'' B_2$:

$(A_1 \otimes A_2) \rhd'' C$:



- for FRP with resources replace $\times$ by $\otimes$

Construction of canonical nonterminating processes

- nullary coherence map of a lax symmetric monoidal functor:

$$1 \to 1 \rhd'' 0$$

- construction of canonical nonterminating processes:

$1 \rhd'' 0$:

- for FRP with resources replace 1 by I

Construction of canonical nonterminating processes

- nullary coherence map of a lax symmetric monoidal functor:

$$I \to I \rhd'' 0$$

- construction of canonical nonterminating processes:

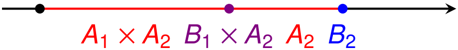
$I \rhd'' 0$:

- for FRP with resources replace 1 by I

Abstract process categories (APCs)

- our existing abstract semantics for FRP without resources
- different, but equivalent, notion of merging, where suffix of longer process is retained:

  $(A_1 \times A_2) \rhd'' D$:



- definition of merging in APCs uses the peculiarities of $\times$ and 1

Theorem

APCs correspond to those of our new categorical models that have the following properties:

- *The symmetric monoidal closed category structure* $(\mathcal{C}, \otimes, I, \multimap)$ *is the cartesian closed category structure* $(\mathcal{C}, \times, 1, \rightarrow)$.
- *The corresponding APC-style merging operator is an isomorphism.*